A NEW EQUIVALENT CIRCUIT OF THE THREE-PHASE INDUCTION MOTOR (CASE STUDIES:CURRENT AND POWER FACTOR OF THE MOTOR)

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ABSTRACT

Characteristics of the three-phase induction motors can be analized by using a conventional equivalent circuit. The parameters of the circuit can be obtained through of several experiment's results in the laboratory such as dc test, no-load test, and blocked-rotor test. All data must be gotten accurately if they are used for predicting the characteristics of the three-phase induction motor. If one data is not gotten accurately, the characteristics of the motor can not be predicted accurately. This study is purposed to give a simple equivalent circuit for analyzing the characteristics of the 3-phase induction motors by using only the nameplate and the no-load test data of the motor. So, the blocked-rotor test and dc test of the motor are not required for the purposed circuit. This study is focused to discuss about the line current and power factor of the three-phase induction motor. The object used in this study was the 3-phase induction motor of 1.5 HP, 380/220V, Y/Δ, 2.7/4.7 A, 4 poles, 50 Hz, 1400 RPM. The results of this study show that the equivalent circuit proposed in this study can be used to predict the characteristics of the three-phase induction motor, especially the input current and power factor of the motor with an accurate rate above 90%.

Keywords: three-phase induction motor, new equivalent circuit, characteristics of the motor.

INTRODUCTION

Three-phase induction motors are widely used in many sections, especially in industrial sectors because these motors are simple and robust construction. These motors are usually produced in high power ratings. These motors are normaly operated by using a three-phase power supply. On specific application conditions, these motors can be operated on single-phase power by installing the capacitor banks to the windings of the motor[1]-[6]. There are many kind other modification of single-phase power supply can be used for operating the three-phase induction motor on single phase supply such as single-phase inverter and PWM control technique [7], [8]. If the three-phase induction motor operated on power supply system, the motor can be analyzed by using characteristics of the motor that monitored during operation. In other condition, the motor can also be analyzed by using an equivalent circuit to predict the characteristics of the motor[9]. The equivalent circuit can be used to predict the characteristics of the motor on various load conditions, even in blocked rotor condition.

Several formulas had been developed in conventional method for calculating the current, and power factor of the 3-phase induction motors. The parameters used in conventional circuit must be obtained from three experimental results such as dc test, no-load test and blocked-rotor test[1], [3], [6], [10]-[15]. Also, some application like automatic rescue devices (ARD) [16] and the reference frame theory to the dynamic analysis of a three-phase induction motor fed from a single-phase supply have been developed [2]. But this method needs some parameters that are received from three experimental result such as dc test, no-load test and blocked-rotor test. If any of the given data is not accurate, the parameter are used for predicting the characteristics of the motors can not be done accurately. So, this study is porposed to give a new equivalent circuit for predicting the characterstis of the three-phase induction motor by using only the data of the no-load test and full-load test of the motor (nameplate data).

EQUIVALENT CIRCUIT AND PREVIOUS WORKS

Three-phase Induction motor is the three-phase alternating current motor that widely used in many aplications. The motor has a strong construction and easy to operate. The naming is derived from the fact that this motor operates by induction the stator magnetic field to the rotor, so that the motor is called the induction motor. The current generated by the rotor (moving parts) is not obtained from a particular source, but is an induced current as a result of the relative difference between the rotation of the rotor and the rotating magnetic field generated by the stator current.

When the stator winding of the three-phase induction motor is connected to a three-phase power supply, the induced voltages will produce rotor currents and torque to the rotor. Then, the rotor will start rotating and reach a steady-state speed 'N' that is less than synchronous speed 'Ns', at which the stator rotating field rotates in the air gap of the motor. The difference between the rotor speed and the stator rotating magnetic field rotates in the air gap by referencing to rotating the magnetic field rotates in the air gap is called the slip 's' that can be discribed as follows.

$$s = \frac{N_s - N}{N_s} \tag{1}$$

Where:
s = slip
N_s = stator speed rotating field rotates in the air gap (RPM)
N = rotor speed rotating (RPM)

Conventional method

The three-phase induction motors are usualy supplied by a balance three-phase voltage source. Therefore, the three-phase induction motors can be analyzed by using a one phase supply equivalent circuit. Figure-1 show a conventional equivalent circuit for analyzing the three-phase induction characteristics when operated on three-phase systems [9]. All data impedances of the motor must be obtained by doing some testing on the motor such as dc-source test, no-load test, and blocked-rotor test. If one data is not true, the characteristics of the motor can not be predicted well [3], [6], [9].

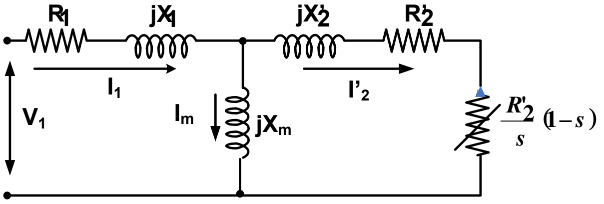

Figure-1. Conventional equivalent circuit of the three-phase induction motor per phase[9].

By referring to Figure-1 can be explained that
V_1 = phase voltage source
R_1 = resistance of the stator windings per phase
X_1 = inductive reactance of the stator windings per phase
R'_2 = resistance of the rotor windings per phase referred to the stator
X'_2 = inductive reactance of the rotor windings per phase referred to the stator
X_m = magnetic reactance of the motor per phase
I_1 = current through the stator windings
I'_2 = current through the rotor windings referred to the stator
s = slip

Induction motors use electrical energy as their energy supply. Some energy are lost in the form of power losses because the resistance component of the stator and rotor windings inside the motor. Power losses in this motor are copper losses, core losses, wind losses and friction losses.

The power description can be calculated mathematically by referring to the equivalent circuit of the motor as shown in Figure-1. From Figure-1 can be calculated the magnitude of the power that crosses the ai gap (P_{ag}) from the stator through to the rotor as follows [9].

$$P_{ag} = (I'_2)^2 \frac{R'_2}{s} = (I'_2)^2 (R'_2 + R'_2 \frac{(1-s)}{s}) \tag{2}$$

Then, the power losses in the rotor circuit (rotor copper loss) is

$$P_2 = (I'_2)^2 R'_2 \tag{3}$$

The mechanical power developed (P_{mech}) by the induction motor can be calculated as follows.

$$P_{mech} = (I'_2)^2 (R'_2 \frac{(1-s)}{s}) \tag{4}$$

When the three powers are compared to the slip (s), it can be made as follows.

$$P_{ag} : P_2 : P_{mech} = 1 : s : (1-s) \tag{5}$$

The current (I_1) and power factor (pf) of the induction motor can be calculated from Figure-1 as follows.

$$I_1 = \frac{V_1}{Z_{tot}} \tag{6}$$

$$pf = \left[\frac{R_{tot}}{Z_{tot}} \right] \tag{7}$$

Where:

$Z_{tot} = R_{tot} + jX_{tot}$ = the total inpedance of the motor circuit
R_{tot} = the total resistance of the motor circuit
X_{tot} = the total raeactance of the motor circuit

All data of the impedance of the induction motor must be obtained accurately from three experimental result such as dc test, no-load test and blocked-rotor test.

Automatic Rescue Devices (ARD)

Automatic Rescue Device (ARD) is a method of sensorless low range speed and parameter identification estimation devoted to lift automatic rescue devices[16]. The equivalent circuit of this method is shown in Figure-2.

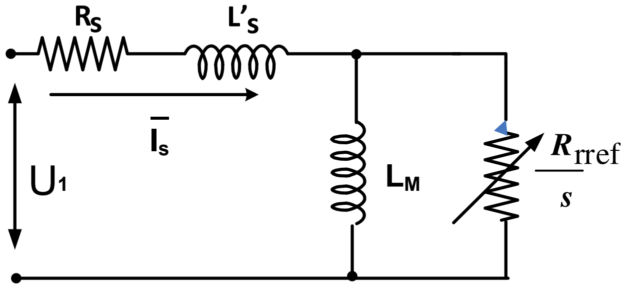

Figure-2. Induction motor sinusoidal steady state equivalent electrical circuit[16].

From the equivalent circuit shown in Figure-2, the equivalent impedance can be determined as follows.

$$Z_{eq} = R_{eq} + jX_{eq} \tag{8}$$

Where:
R_{eq} = equialent motor resistance
X_{tot} = equialent motor reactance
Z_{tot} = equialent motor impedance

From equation (8) can be explained that the equivalent resistance (R_{eq}) and the equivalent reactance (X_{eq}) are

$$R_{eq} = R_s + \frac{\omega^2.s.L_M^{'}.T_r}{1+(s.\omega.T_r)^2} \tag{9}$$

$$X_{eq} = \omega.L'_s + \frac{\omega.L_M^{'}}{1+(s.\omega.T_r)^2} \tag{10}$$

Where:
R_s = stator resistance
L_s = stator transient inductance
L_M = referred magnetizing inductance
T_r = rotor time constant
ω = stator angular frequency

The active (P) and reactive (Q) power of induction motor are

$$P = (\overline{I_s})^2.R_{eq} \tag{11}$$

$$Q = (\overline{I_s})^2.X_{eq} \tag{12}$$

Where:
$\overline{I_s}$ = stator spase phasor current

Then, the active power delivered to the rotor (P_{sr}) and the reactive power delivered to the rotor (Q_{sr}) are respectively as follows:

$$P_{sr} = P - (\overline{I_s})^2.R_{eq} \tag{13}$$

$$Q_{sr} = Q - (\overline{I_s})^2.\omega.L_M \tag{14}$$

The referred rotor resistance (R_{rref}) can be determine as follows:

$$R_{rref} = \frac{L_M^{'}}{T_r} \tag{15}$$

By substituting equation (15) in (9) and (10) then are obtained as follows:

$$R_{eq} = \frac{P}{\left|\overline{I}_s\right|^2} = R_s + \frac{\omega^2.s.L_M^2 / R_{rref}}{1+(s.\omega.L_M / R_{rref})^2} \tag{16}$$

$$X_{eq} = \frac{Q}{\left|\overline{I}_s\right|^2} = \omega.L'_s + \frac{\omega.L_M}{1+(s.\omega.L_M / R_{rref})^2} \tag{17}$$

By comparing equations (13) and (14) respectively with equations (16) and (17), then both the active and reactive power delivered to the rotor from the stator are

$$P_{sr} = \left|\overline{I}_s\right|^2.\frac{\omega^2.s.L_M^2 / R_{rref}}{1+(s.\omega.L_M / R_{rref})^2} \tag{18}$$

$$Q_{sr} = \left|\overline{I}_s\right|^2.\frac{\omega.L_M}{1+(s.\omega.L_M / R_{rref})^2} \tag{19}$$

By dividing equation (18) by (19) then will be obtained as follows:

$$\frac{P_{sr}}{Q_{sr}} = \frac{\omega.s.L_M}{R_{rref}} \tag{20}$$

Then, the equation (20) can be substituted in the second term of the denominator in equation (19) giving the value of the referred magnetizing inductance as follows:

$$L_M = Q_{sr}\frac{1+\left(\dfrac{P_{sr}}{Q_{sr}}\right)^2}{\omega.\left|\overline{I}_s\right|^2} = \frac{P_{sr}^2+Q_{sr}^2}{\omega.\left|\overline{I}_s\right|^2.Q_{sr}} = \frac{A_{sr}^2}{\omega.\left|\overline{I}_s\right|^2.Q_{sr}} \tag{21}$$

Where:
A_{sr} = apparent power delivered to the rotor

By substituting equation (21) in (20), the expression of the motor slip (s) is obtained as follows:

$$s = R_{rref}\frac{P_{sr}}{\omega.L_M.Q_{sr}} = R_{rref}\frac{P_{sr}.\left|\bar{I}_s\right|^2}{A_{sr}^2} \tag{22}$$

The speed can be calculated as follows:

$$\Omega_r = (1-s)\frac{\omega}{p} \tag{23}$$

Where:

Ω_r = rotor angular speed

Previous works and proposed menthod

The 3-phase induction motors have multiple windings placed in the slots. The equivalent circuit model for one coil of the motors can be drawed as shown in Figure-3[17].

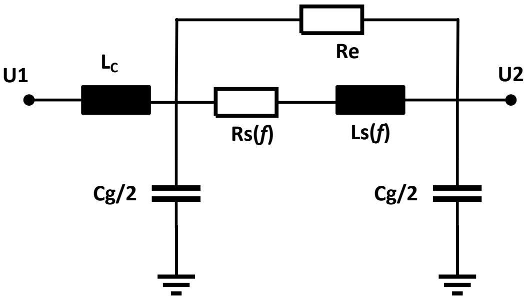

Figure-3. The equivalent circuit model for one coil of the motor[17].

By referring to Figure-3 can be explained that

Rs(*f*) = resistance cover one coil of the winding
Ls(*f*) = self-inductance cover one coil of the winding
Re = iron losses
Cg = capacitance to the ground
U1, U2 = voltage between one coil

With reference to Figure-1 and Figure-3, a new equivalent circuit for the 3-phase induction motor can be created as shown in Figure-4. This Figure is a another parallel equivalent circuit from Figure-1 with combine to Figure-3.

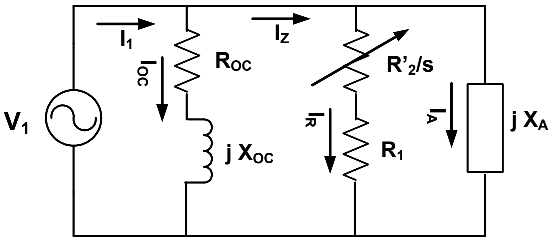

Figure-4. A new equivalent circuit for analyzing the characteristics of the three-phase induction motor (proposed method).

By referring to Figure-4 can be explained that

R_{OC} = no load test resistance of the motor's windings per phase
X_{OC} = no load test reactance of the motor's windings per phase
X_A = additional reactance of the motor (usually capacitive reactance)
I_1 = phase current through to the motor at full load
I_{OC} = no load test current through the windings of the motor
I_R = the additional current through the resistance of the motor
I_A = the additional current through the additional reactance
V_1 = phase voltage of the motor

Figure-4 is a simple equivalent circuit model that can be used for analyzing the characteristics of the 3-phase induction motors when operated on the 3-phase power system. By using Figure-4 we only need the data of no-load test and full-load test (nameplate data) of the motor. So, we need not DC test and blocked-rotor test for analyzing the characteristic of the motor. By referring to Figure-4, the magnitude of the no load test impedance (Z_{OC}) of the motor can be written as follows:

$$Z_{OC} = R_{OC} + jX_{OC} \tag{24}$$

There are two ways to determine the impedance of no-load test (Z_{OC}) of the motor depending on the motor windings connection system. When the motor's windings are connected in 'Delta' connection standard, the magnitude of the 'Z_{OC}' for delta standard ($Z_{OC(\Delta)}$) becomes:

$$Z_{OC(\Delta)} = \frac{\sqrt{3}.V_{LL(\Delta)}}{I_{L(OC)}} \tag{25}$$

Then, when the motor's windings are connected in 'Wye' connection standard, the magnitude of the 'Z_{OC}' for wye standard ($Z_{OC(Y)}$) becomes:

$$Z_{OC(Y)} = \frac{V_{LL(Y)}}{\sqrt{3}.I_{L(OC)}} \tag{26}$$

Where:

V_{LL} = line to line voltage source on no load test
$I_{L(OC)}$ = line current on no load test
$Z_{OC(Y)}$ = Z_{OC} for wye standard motor
$Z_{OC(\Delta)}$ = Z_{OC} for delta standard motor

The full-load current of the motor (nominal current of the motor) can be obtained from full-load test data or from the data written on the nameplate of the motor. If the motor is operated on 'Delta (Δ)' connection standard, then I_1 from Figure-4 is equal to the full load current divided by 1.73205. Then, if the motor is operated on ‘Wye (Y)’ connection standard, I_1 from Figure-4 is equal to the full load current (nominal current). So, the line current magnitude of the 3-phase induction motors for ‘Δ’ and ‘Y’ connection standard can be writen as follows:

$$I_{L(\Delta)} = \sqrt{3}.I_1 \tag{27}$$

$$I_{L(Y)} = I_1 \tag{28}$$

The currents ‘I_Z’, ‘I_R’ and ‘I_A’ from the Figure-4 then can be written as follows:

$$I_Z\angle\theta = I_1\angle\phi - I_{OC}\angle\beta \tag{29}$$

$$I_R = I_Z.\cos(\theta) \tag{30}$$

$$I_A = I_Z.\sin(\theta) \tag{31}$$

Where:

ϕ = the angle of the full load current
β = the angle of the no-load current
θ = the angle of the current ‘I_Z’

The magnitude of R_1 , R'_2 and X_A from the Figure-4 then can be calculated as follows:

$$X_A = \frac{V_1}{I_A} \tag{32}$$

$$R_1 + \frac{R'_2}{s} = \frac{V_1}{I_R} \tag{33}$$

By assuming $R_1 = R'_2$, then will be obtained as follows:

$$R_1 = R'_2 = \frac{V_1}{(1+1/s).I_R} \tag{34}$$

For certain conditions, the data of power factor of the motor is not given on the motor nameplate, so that the motor should be operated directly under full load condition to find the power factor of the motor. By referring to Anthony’s research about the capacitance of the run capacitor 'Cr_y' that used to operate the 3-phase induction motor on single phase supply, the full load power factor of the motor can be calculated accurately as shown in formula of equation (35) to equation (38) [5]. The 'Cr_y' is used for operating the three-phase induction motor on single phase supply that can be calculated as follows[5].

$$Cr_y = k\frac{I_L}{(12{,}5664)(f).(V_{LN})} \tag{35}$$

Where:

I_L = line current of the 3-phase induction motor
V_{LN} = line to neutral of the single phase supply
Cr_y = capacitance of the run capacitor for wye connection standard
k = constant factor

When the 'k' is 1 (for the motor is operated with a close to 3-phase rating), the run capacitor in equation (35) should have ‘reactive power (Q_C)’ as follows.

$$Q_C = \omega.Cr_y.(V_C)^2 = 2.V_{LN}.I_L \tag{36}$$

Where:

V_C = volltage on capacitor
ω = 2.π.f
f = frequency

If the full-load reactive power of the motors 'Q_M' is compared against to the 'Q_C' from the equation (36), the magnitude of '$Q_M = 0.983\ Q_C$' (referring to the rotor speed standard of the motor) or '$Q_M = 0.997\ Q_C$' (referring to the nominal current of the motor). Therefore, the magnitude of 'Q_M' would be defined as follows:

$$Q_M = 0.99Q_C \tag{37}$$

Then, the power factor of the motor can be calculated as follows.

$$\cos\phi = \cos\left(\tan^{-1}\phi\right) \tag{38}$$

Where:

$$\tan\phi = \frac{Q_M}{S_M} \tag{39}$$

and,

$$S_M = \sqrt{3}.V_{LL}.I_L \tag{40}$$

$\cos\phi$ = power factor of the motor
S_M = apparent power of the motor

METHODOLOGY

The motor used in this study was the 3-phase induction motor of 1.5 HP, 380/220V, Y/Δ, 2.7/4.7 A, 4 poles, 50 Hz, 1400 RPM. The study is focused about for calculating the line current and power factor of the motor by using the equivalent circuit as shown in Figure-4. The equivalent circuit and the formulas created in this study will be compared with the experiment results in the laboratory. The motor is operated by using 'Wye' connection standard winding. The circuit equipment and accessories used for operating the 3-phase induction motor is shown in Figure-5.

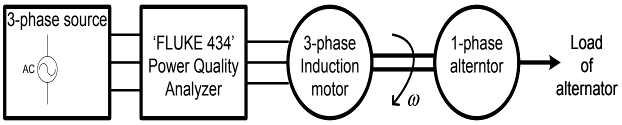

Figure-5. Circuit equipment and accessories used in the laboratory.

RESULT AND DISCUSSIONS

The results of this study are given in Table-1 and Table-2 as below.

Table-1. The input current during operation against the calculation results by using the formulas created.

No.	I_L (exp)	I_L (c)	**Error** (%)	**Nr** (RPM)
1	1.829	1.902	-4.013	1481
2	1.872	1.874	-0.118	1476
3	1.949	1.924	1.303	1469
4	1.998	1.999	-0,025	1460
5	2.113	2.138	-1.193	1452
6	2.206	2.217	-0.490	1444
7	2.323	2.253	3.026	1434
8	2.436	2.413	0.944	1423
9	2.610	2.676	-2.513	1403
10	2.755	2.883	-4.632	1386

Where:

I_L (exp) = the current from motor operation data condition

I_L (c) = the current from the calculation results by using the formulas created

Error (%) = percentage error calculation results when compared against to the results of the experiment

Nr(RPM) = rotor speed (Rotation per minutes)

Table-2. The power factor during operation against the calculation results by using the formulas created.

No.	**PF** (exp)	**PF** (c)	**error** (%)	**Nr** (RPM)
1	0.43	0.364	15.395	1481
2	0.48	0.391	18.479	1476
3	0.54	0.444	17.704	1469
4	0.58	0.506	12.793	1460
5	0.62	0.559	9.823	1452
6	0.66	0.600	9.030	1444
7	0.70	0.644	8.057	1434
8	0.73	0.688	5.767	1423
9	0.75	0.749	0.133	1403
10	0.77	0.789	-2.403	1386

Where:

PF (exp) = power factor from motor operation data condition

PF (c) = power factor from the calculation results by using the formulas created

Characteristics of the line current and power factor of the motor are shown in Table-1 and Table-2. By refering to Table-1 can be seen that the error of the calculation results when compared to the experimental results are below 5%. So, it can be said that the equivalent circuit and the formulas given have hight accuracy for predicting the line current of the motor (accuracy level is about 95%). Then, by referring to Table-2, the formulas created have good result for calculating the power factor of the motor at the rotor speed below 1452 RPM (the error is below 10%). Therefore, the formulas created on the circuit are suiTable for analyzing the power factor of the motor for hight load condition (accuracy level is about 90%). If both tables are converted into graphs, the results can be seen in Figure-6 and Figure-7.

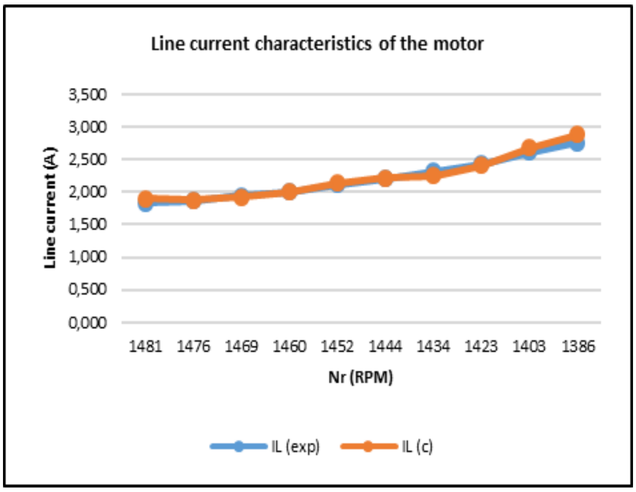

Figure-6. Comparison characteristic between current 'IL (exp) against to current 'IL (c)'.

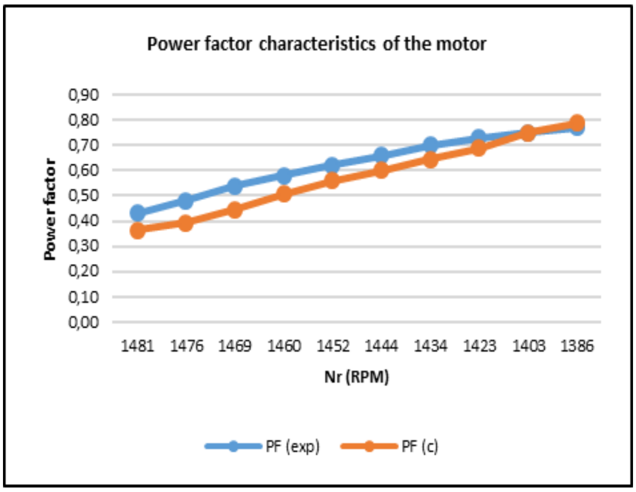

Figure-7. Comparison characteristic between power factor 'PF (exp) against to power factor 'PF (c)'.

From both Figure-6 and Figure-7 can be seen that the characteristics of the motor between calculation result and operation result are similar. So, the equivalent circuit with the formula created is suitable for predicting the characteristics of the line current and the power factor of the motor.

CONCLUSIONS

The results of this study show that the equivalent circuit and formulas created have high accuracy. The errors of the calculation results when compared to the experimental results are below 5%. So, the formulas created have hight accuracy for predicting the line current of the motor (accuracy level is about 95%). Then, the formulas created also have a good result for calculating the power factor of the motor especially at the rotor speed below 1452 RPM (high load condition), where the errors are below 10%. Therefore, the formulas created on the circuit are also suitable for analyzing the power factor of the motor for hight load condition (accuracy level is about 90%). The current and power factor characteristics of the calculation result are similar with the operating characteristics of the motor.

ACTKNOWLEDGEMENTS

We would like to thank the team at the laboratory of electrical engineering of the 'Institute of Technology Padang (Institut Teknologi Padang)' whose have helped this study run smoothly. We would also like to thank for the 'Directorate General of Human Resources for Science, Technology and Higher Education of Indonesia' that has funded this research by agreement No. 1275/27.O10.4.2/PN/2016.

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